

UNIVERSITY OF ILLINOIS  
AT URBANA-CHAMPAIGN

# Instrumental Variables



[illinois.edu](http://illinois.edu)

# Outline for the Session

1. When is the treatment exogenous?
2. When is the treatment endogenous?
3. What is an IV and how does it work?
4. Operationalizing IVs
5. Source of IVs



# When is the Treatment Exogenous?



# Exogenous Treatment

- Exogeneity of a treatment relies on two assumptions:
  - SUTVA
  - Ignorability/Unconfoundedness:  $(Y_i^T, Y_i^C) \perp T$
- Random assignment of treatment insures that treatment is independent of outcome. Thus, treatment and control groups are the same and any selection bias is erased



# When is the Treatment Endogenous?



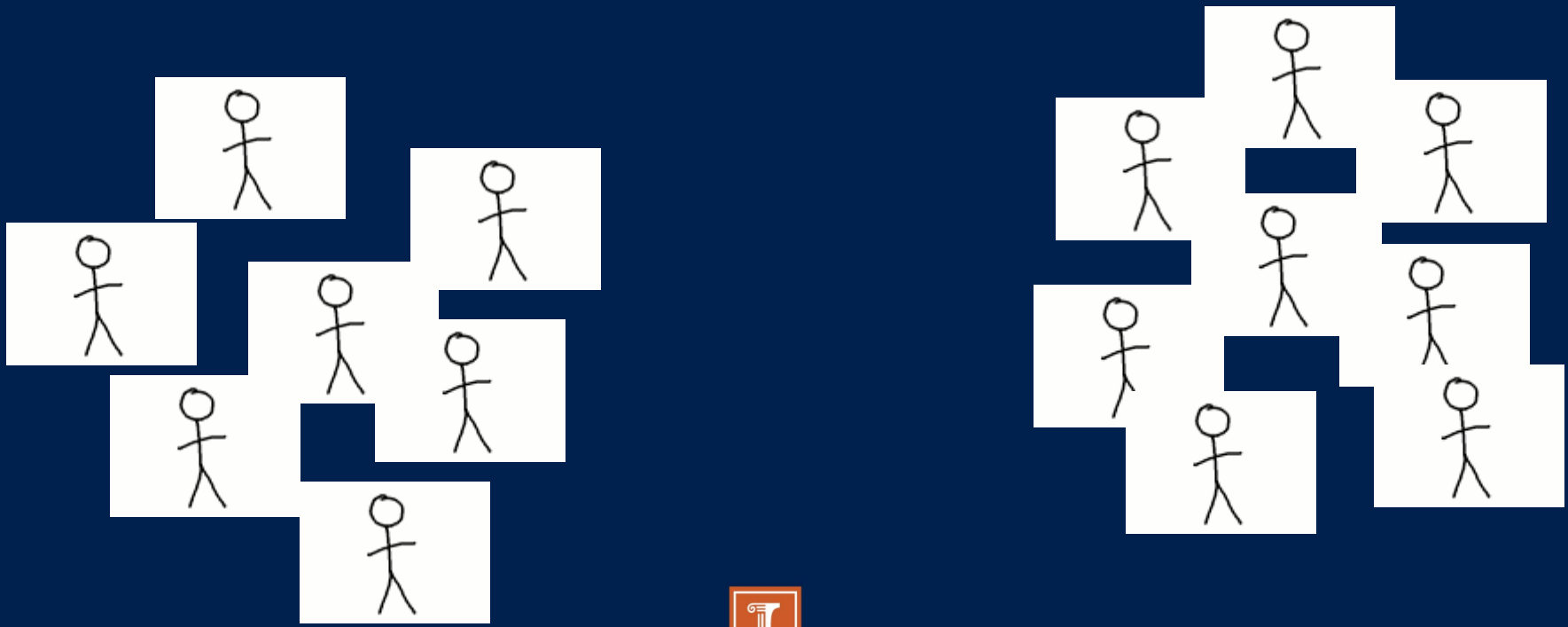
# The Potential Outcomes Approach

- When our randomized design is either an encouragement design or we have imperfect compliance
  - In this case, actual treatment ( $T$ ) is distinct from the variable that is randomly manipulated ( $Z$ )
  - We can then define the compliance type of an individual
  - The type of an individual describes the level of treatment that an individual would receive given each value of the instrument. So we have  $T_i(Z)$



# Example

- Encouragement design



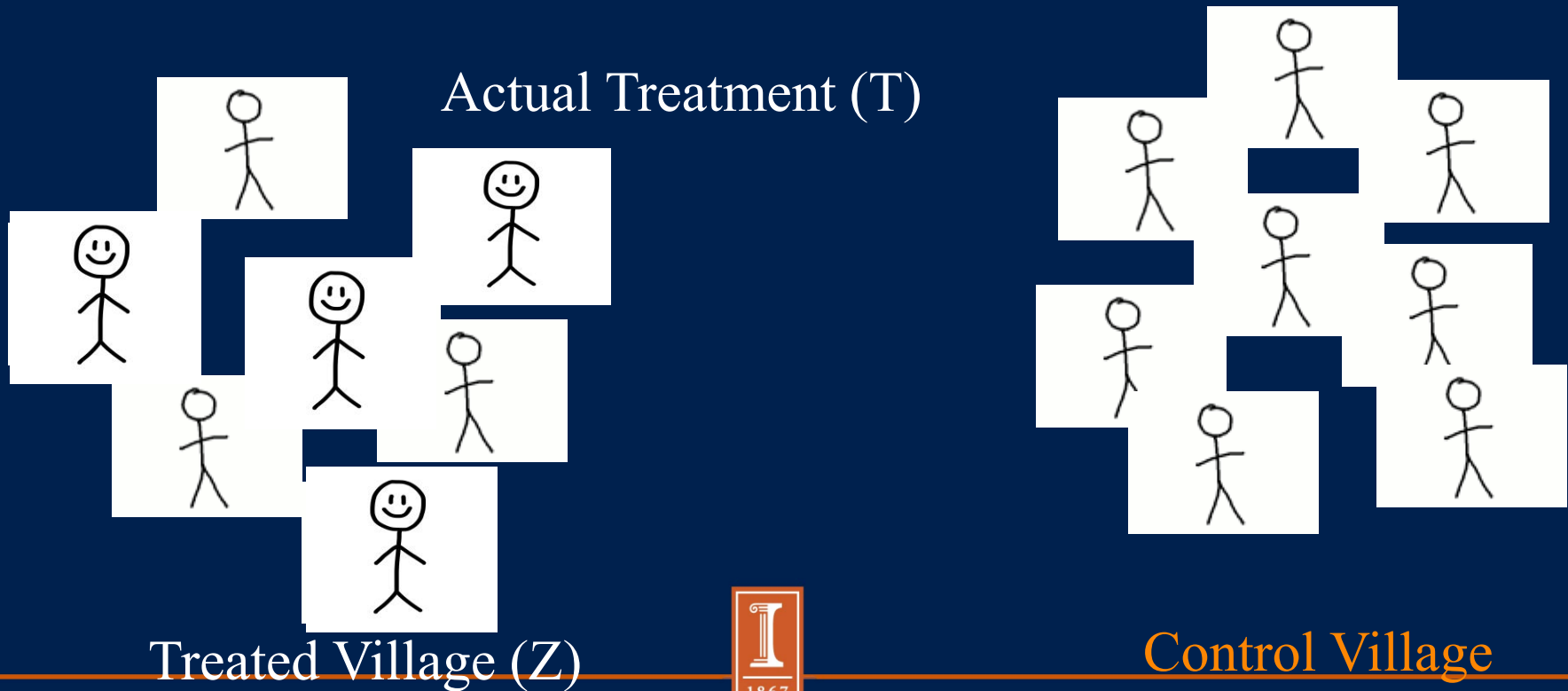
Treated Village (Z)

Control Village



# Example

- Only some people adopt





# The Potential Outcomes Approach

- Four types of individuals  $T_i(Z)$ 
  - Never-takers:  $T_i(0) = T_i(1) = 0$
  - Compliers:  $T_i(0) = 0, T_i(1) = 1$
  - Defiers:  $T_i(0) = 1, T_i(1) = 0$
  - Always-takers:  $T_i(0) = T_i(1) = 1$

		$Z_i$	
		0	1
$T_i$	0	Never-takers/Compliers	Never-takers
	1	Always-takers	Always-takers/Compliers



# The Potential Outcomes Approach

- Given the observed data  $(Z_i, T_i, Y_i)$  we cannot tell the difference between
  - A complier and an always-taker
  - A complier and an never-taker
- What we require is some additional assumptions that will allow us to identify the complier from the always-taker



# The Endogenous Regressor Approach

- When random assignment does not exist and we must use observational data
  - Treatment assignment may not be independent of outcome
  - Ignorability/Unconfoundedness assumption no longer holds
- In the regression context:  $Y_i = \alpha + \beta T_i + \epsilon_i$ 
  - We can no longer assume  $Cov(T_i, \epsilon_i) = 0$
  - This violates a principal assumption of OLS



# Case 1: Treatment Assignment is Non-Random

- This is endogeneity due to targeting or program placement
- If targeting or program placement is based on observables the solution is easy
  - We can just include the relevant covariates
  - $Y_i = \alpha X_i + \beta T_i + \epsilon_i$
  - By including the relevant covariates in  $X_i$  we can ensure that treatment, conditional on those observables, is no longer correlated with the error term



## Case 2: Treatment Assignment is Non-Random and Affected by Unobservables

- This is endogeneity due to unobserved heterogeneity
- Including covariates no longer solves the problem
  - Since treatment is dependent on something we cannot observe, that missing or omitted variable ends up in the error term
  - $Cov(T_i, \epsilon_i) \neq 0$
  - In this situation we require a variable that can *instrument for the endogenous treatment* and break correlation between the treatment and the error term



## Summary and Discussion

- In both the Potential Outcome Approach and in the Endogenous Regressor Approach we require a set of assumptions and relevant data that will allow us to identify the causal effect.
  - These assumptions are called *Identification Assumptions* and the relevant data are called *Instrumental Variables*
- What are examples of treatment assignment that is not independent of outcomes?



# What is an IV and How Does it Work?



# Identification Assumptions

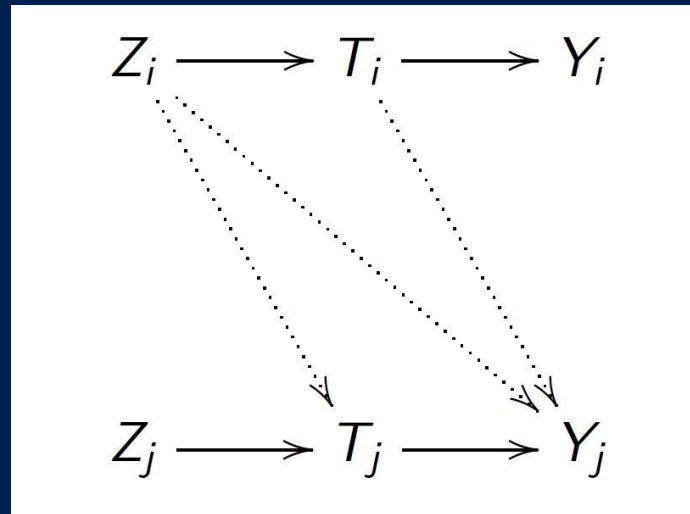
1. SUTVA
2. Exogeneity of the instrument
3. Non-zero average effect of  $Z$  on  $T$
4. Monotonic effect of  $Z$  on  $T$





# 1. SUTVA

- $Z_i$  does not affect  $T_j$  and  $Y_j$  and  $T_i$  does not affect  $Y_j$  for all  $i \neq j$  (non-interference)



- The value of my instrument or the status of my treatment does not affect your treatment or your outcome



## 2. Exogeneity of the Instrument

- All potential outcomes are independent of the instrument

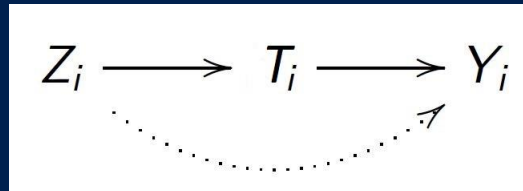
$$(Y_i(0), Y_i(1), T_i(0), T_i(1)) \perp Z_i$$

- This assumption is really made up of two assumptions



## 2. Exogeneity of the Instrument

- 2A. Ignorability/Unconfoundedness of  $Z_i$ 
  - Instrument is not correlated with any unobservables that affect the outcome so that its effect on the outcome and treatment received can be consistently estimated
- 2B. Exclusion Restriction



- There is no direct effect of the instrument on the outcome. Any effect of  $Z_i$  on  $Y_i$  must be through the treatment  $T_i$
- $Cov(Z_i, \epsilon_i) = 0$



### 3. Non-Zero Average Effect of $Z$ on $T$

- Instrument must be correlated with treatment

$$\text{Cov}(Z_i, T_i) \neq 0$$



## 4. Monotonicity

- Increasing the level of the instrument does not decrease the level of the treatment

$$T_i(1) \geq T_i(0) \quad \forall i$$

- This amounts to their being no defiers



# Instrumental Variables

- A variable that is a valid instrument for the endogenous treatment is any variable that satisfies the above identifying assumptions
- By using an IV we are able to isolate the part of the treatment variable that is independent of other unobserved characteristics affecting the outcome



## One drawback

- Using an IV, we are gaining unbiasedness but losing some efficiency
- In a simple 2-variable case:

$$\text{Var}(\hat{\beta}_{iv}) = \frac{s^2}{n\text{Var}(X)} \frac{1}{\text{corr}(X, Z)^2}$$

$$\text{Var}(\hat{\beta}_{ols}) = \frac{s^2}{n\text{Var}(X)}$$

Why not have a Z that is perfectly correlated with X?



# Recap

- But we don't want the correlation between X and Z to be too small
- Recall: 
$$\widehat{\beta}_{iv} = \beta + \frac{\text{Cov}(u_i, Z_i)}{\text{Cov}(T_i, Z_i)}$$





# Operationalizing IVs



## Two-stage least squares (2SLS)

- First, regress treatment on instrument and other exogenous variables

$$T_i = \gamma Z_i + \phi X_i + u_i$$

- Second, calculate the predicted treatment from this regression

$$\hat{T}_i = \hat{\gamma} Z_i + \hat{\phi} X_i$$



## Two-stage least squares (2SLS)

- Third, replace  $T_i$  with its predicted value  $\hat{T}_i$  in to create the reduced form regression equation

$$Y_i = \alpha X_i + \beta(\hat{\gamma}Z_i + \hat{\phi}X_i) + \epsilon_i$$

- In practice we estimate this in a single step

$$Y_i = \alpha X_i + \beta\hat{T}_i + [\epsilon_i + \beta(T_i - \hat{T}_i)]$$

- Note that the standard errors will be wrong



## Recap of IV and 2SLS Lingo

- Endogenous variables
  - Independent variables to be instrumented – is correlated with the error term
- Treat an independent variable as endogenous
  - To instrument a variable, meaning to replace it with its fitted values in the second stage of the 2SLS procedure
- Exogenous variables
  - Independent variables (and IVs) that are uncorrelated (orthogonal) with the error term
- Use IV commands to ensure SE are correct



## Calculating the ATE

- If we had perfect randomization then we could run the following regression

$$Y_i = \beta T_i + \epsilon_i$$

- Then the Average Treatment Effect is just

$$\beta = ATE$$



## Calculating the LATE

- But our IV estimate of the treatment effect is:

$$\widehat{\beta}_{IV} = \beta + \frac{Cov(u_i, Z_i)}{Cov(T_i, Z_i)}$$

- This is only a local effect or LATE because it's the effect of  $T_i$  on  $Y_i$  for the subpopulation of compliers, and not the whole



# Specification tests



# Durbin-Wu-Hausman Test

- One should test for endogeneity of the treatment
  - First, regress  $T_i$  on  $Z_i$  and other exogenous covariates,  $X_i$ , and obtain the residuals,  $\hat{u}_i$ 
    - *These residuals reflect all unobserved heterogeneity affecting treatment not captured by the instruments*
  - Second, regress  $Y_i$  on  $X_i$ ,  $Z_i$ , and  $\hat{u}_i$ 
    - *If the coefficient on  $\hat{u}_i$  is significant, unobserved characteristics jointly affecting  $T_i$  and  $Y_i$  are significant then the null that  $T_i$  is exogenous is rejected.*
- Note that this test assumes that the IV is valid and is not a test for the validity of the IV





# Wu-Hausman Statistic

A.k.a. Hausman specification test

$$H = (\hat{\beta}_{ols} - \hat{\beta}_{iv})' (Var(\hat{\beta}_{iv}) - Var(\hat{\beta}_{ols}))^{-1} (\hat{\beta}_{ols} - \hat{\beta}_{iv})$$

- Assumes IV is unbiased
- Compares degree of bias to efficiency loss



# Weak Instruments

- “Cure can be worse than the disease”
- We don’t want the correlation between  $T_i$  and  $Z_i$  to be too small

$$\widehat{\beta}_{iv} = \beta + \frac{\text{Cov}(u_i, Z_i)}{\text{Cov}(T_i, Z_i)}$$

Test predictive power in the first stage

F-stat of instrument(s)

For critical values, see (Stock and Yogo 2005)



# Sargan-Hansen Test for Overidentification

- No test exists to determine if the IV satisfies the exclusion restriction
  - Justification can only be made through direct evidence of how the program and participation evolved
- One can test for overidentifying restrictions
  - First, estimate the structural equation by 2SLS and obtain  $\hat{\epsilon}_i$
  - Second, regress  $\hat{\epsilon}_i$  on  $X_i$  and  $Z_i$  and obtain the  $R^2$
  - With a null of no correlation between  $X_i$  and  $\hat{\epsilon}_i$ , test if  $nR^2$  is greater than the critical value. If so then at least one of the instruments is not exogenous



# Source of IVs



# What Qualifies as a Good IV?

- So, where can you find a good instrument?

“Good instruments come from a combination of institutional knowledge and ideas about the process determining the variable of interest”

-Angrist and Pischke

*Mostly Harmless Economics*



# What Qualifies as a Good IV?

- An IV can be external or randomly assigned but that does not mean the IV is exogenous
- External
  - A variable whose value is set outside of the causal system
  - It is “as good as” randomly determined
- Exogenous
  - A variable that is uncorrelated with (orthogonal to) the error term
  - Satisfies *both* 2A (unconfoundedness/ignorability) *and* 2B (the exclusion restriction)



## Discussion of IV Quality

- Giles and Yoo, 2007, ReStat
  - $Y_i$ : consumption;  $T_i$ : household migrant/network
  - $Z_i$ : Rainfall shocks from distant past
- Burgess et al., 2012, QJE
  - $Y_i$ : deforestation;  $T_i$ : local government permit to log
  - $Z_i$ : subdividing of local governments
- Di Falco and Veronesi, 2013, Land Econ
  - $Y_i$ : Net revenue;  $T_i$ : Adaptation strategy
  - $Z_i$ : Access to information sources like extension



## Critique: Deaton (2010)

- Instruments: exogenous versus external
  - E.g. rail stations and poverty (river; earthquake)
  - Irrigation dams (land gradient)
  - Child class size; some people don't stay treated (heterogeneous response to instrument)
  - Intent to Treat vs Treatment. Really evaluating those communities/individuals who were induced to change. May not be representative of all communities





## Critique: Deaton (2010)

- Important question is not ‘if it works’ but ‘why (or when and where) it works’
  - RCT:
    - *relies on mean; what if distributions between T and C differ?*
    - *Heterogeneity (one guy wins big, everyone else loses)*
    - *Scaling up? (general equilibrium effects)*
    - *Generalizability – is it meaningful?*
    - *Controlling for other things can be a problem with heterogeneity*
- Tests of theory versus test of programs (help with external validity)



# Types of Treatment Effects



# Which Treatment Effect to Measure?

- There are a number of different ways to measure the effect of treatment
  - ATE: Average Treatment Effect
  - ATT: Average Treatment Effect on the Treated
  - ATUT: Average Treatment Effect on the Untreated
  - ITT: Intent to Treat Estimate
  - LATE: Local Average Treatment Effect
  - MTE: Marginal Treatment Effect



# Which Treatment Effect to Measure?

- Different treatment effects are an average over parts of the distribution of impacts
  - The ATE averages over the entire distribution
  - The ATT averages over the distribution of impacts for those allocated to the treatment
  - The LATE averages over the distribution of impacts for those who switch into the treatment as the result of a change in an some instrument



# Which Treatment Effect to Measure?

- These all represent an aggregation over different margins
  - As such, they are not comparable to each other
- As a unifying measurement Heckman and Vytlacil (2005) defined the MTE
  - The MTE is the effect of the treatment on the marginal individual entering treatment



# Basic Requirements

- What is needed to measuring the treatment effect?
  - Assumptions
    - *SUTVA*
    - *Ignorability/Unconfoundedness*
  - Data
    - *Observations on outcomes for those who were treated*
    - *Observations on outcomes for some constructed control group*
- Without observations from treated individuals and from some sort of control group we cannot measure the effect of the treatment!



# What We Can Never Measure

- Notice that all our measurements of treatment effects are averages
  - The Fundamental Problem of Causal Inference
  - We do not observe subject in simultaneous treated and untreated states
- So, we can never determine the effect of the treatment on an individual
  - We can only ever determine the average effect of the treatment or the effect of the treatment on an average individual

